In the present project, following our research plan, we have done research and established a number of significant results in the following four areas:

(I) Set-theoretic topology (compact spaces, scattered spaces, cardinal functions, resolvability)
(II) Descriptive set-theory
(III) Combinatorics
(IV) Real analysis and measure theory

We presented our results in 45 papers almost all of which appeared or will appear in the leading international journals of these fields (6 of these papers have been submitted but not accepted as yet). Our research group consisted of 8 people, one of us – J. Gerlits – unfortunately passed away in 2008. We also participated at a large number of international conferences, four of us (Elekes, Juhász, Mátrai, Soukup) as plenary and/or invited speakers at many of these. We now give an overview of our results.

I. Set-theoretic topology

– compact spaces

The study of the important class of **compact spaces** traditionally has occupied a central place in our investigations. In 13. we gave a considerable strengthening of the classical (more than 70 year old) Čech–Pospiǐsil theorem which states that if every point of a compact T_2 space X has character $\geq \kappa$ then $|X| \geq 2^\kappa$. We showed that under the same assumption the compact space X cannot even be covered by fewer than 2^κ discrete subspaces.

In 9. we studied and gave partial answers to questions of Arhangelskii and Buzyakova concerning compacta in which there is no convergent sequence whose length is an uncountable regular cardinal. We call these K-compacta, and the related but wider class of compacta in which the character of every point has countable cofinality AB-compacta. The most intriguing question of Arhangelskii and Buzyakova, namely if every K-compactum is first countable, is still open in ZFC, but we proved in 10. that if the continuum $\mathfrak{c}$ is less than $\aleph_\omega$ then even all AB-compacta are first countable. We also proved in ZFC that the cardinality of any AB-compactum is at most $2^{<\mathfrak{c}}$.

The investigations that we started in 9. led to the quite extensive general study of the convergence and character spectra of compacta in 36. The convergence spectrum $cS(X)$ of a space X is the set of all sizes of converging (one-to-one) sequences in X, while the character spectrum $\chi S(X)$ is the set of all characters of (non- isolated) points in subspaces of X. For compacta (that we are really interested in) we always have $cS(X) \subset \chi S(X)$. Here is a selection of the results of 36. (X is always a compactum):

(1) If $\chi(X) > 2^\omega$ then $\omega_1 \in \chi S(X)$ or $\{2^\omega, (2^\omega)^+\} \subset \chi S(X)$.
(2) If $\chi(X) > \omega$ then $\chi S(X) \cap [\omega_1, 2^\omega] \neq \emptyset$.
(3) If $\chi(X) > 2^\kappa$ then $\kappa^+ \in cS(X)$, in fact there is a converging discrete set of size κ^+ in X.
(4) If we add λ Cohen reals to a model of GCH then in the extension for every $\kappa \leq \lambda$ there is X with $\chi S(X) = \{\omega, \kappa\}$. In particular, it is consistent to have X with $\chi S(X) = \{\omega, \aleph_\omega\}$. Note that this X is a non-first countable AB-compactum but not a K-compactum: $\omega_n \in cS(X)$ for every $n < \omega$.
(5) If all members of $\chi S(X)$ are limit cardinals then

$$|X| \leq (\sup\{|\overline{S}| : S \in [X]^\omega\})^\omega.$$

(6) It is consistent that 2^ω is as big as you wish and there are arbitrarily large X with $\chi S(X) \cap (\omega, 2^\omega) = \emptyset$.

The last item (6) shows that the character spectrum of a non-first countable compactum may (consistently) omit ω_1, the first uncountable cardinal, but it was left open in 36. if the convergence spectrum can do that. Item (3) implies that this may only happen if $\chi(X) \le \mathfrak{c} = 2^\omega$. This problem turned out to be very hard and it took us in 24. a lot of work to construct, with a very complicated forcing argument, a compactum X such that $cS(X) = \{\omega, \omega_2\}$. So far, this is the only known (consistent) example of a non-first countable compactum whose convergence spectrum omits ω_1.

Answering a question of Tkachuk, we proved in 14. the following surprising result: The ωth power of every compactum is d-separable, i.e. it has a σ-discrete dense subset. (On the other hand, it is known that there is a compactum no finite power of which is d-separable.) The proof of this hinges on the following result that is interesting in itself: The square X^2 of every compactum X contains a discrete subset of size $d(X)$, the density of X. The latter statement was sharpened in 30. to obtain the following result that is of interest for functional analysts: Every compactum X possesses a bidiscrete system of size $d(X)$. A bidiscrete system for X is a set of pairs $\{(x_\alpha, y_\alpha) : \alpha < \kappa\} \subset X^2$ such that there are continuous real functions $\{f_\alpha : \alpha < \kappa\} \subset C(X)$ with the property that f_α separates the pair (x_α, y_α) but does not separate any of the other pairs. A bidiscrete system thus provides for the Banach space $C(X)$ a so-called nice biorthogonal system.

A celebrated reflection theorem of Dow states that if every subspace of cardinality ω_1 of a compact space X is metrizable then so is X. Arhangelskii asked if this is also true for locally compact spaces and in 34. we proved that the answer to this question is independent of ZFC. More importantly, we introduced in 34. a reflection principle, we called it Fodor-type reflection principle, that is much weaker than Fleissner's Axiom R but still implies most of its known consequences, in particular (the consistency of) the affirmative answer to Arhangelskii's question. The topological methods used to establish this can also be applied under various other circumstances. Thus another interesting result from 34., proved in ZFC, is that metrizability has the singular compactness property in the class of locally separable and countably tight spaces. That is, if every subspace of such a space X of size smaller than $|X|$ is metrizable so is X, provided that $|X|$ is a singular cardinal.

While it is well-known that the (Vietoris) hyper space $H(X)$ of a space X is compact if and only if X is, it has been a celebrated open problem to find a criterion for the countable compactness of $H(X)$. In 5. we found a sufficient condition for this that seems to be very close to being also necessary, though we could not prove this. However, our condition is very effective in the sense that it significantly improved a number of earlier results. We also succeeded in giving consistent examples showing that even such „nice" topological properties as first countability or normality do not suffice to yield the countable compactness of $H(X)$ from that of X.

Finally, we mention here the results of 15. because of its connections with countable compactness. An old conjecture of Nagata stated that every M-space is homeomorphic to a closed subspace of the product of a metric space with a countably compact space. This was refuted by Burke and van Douwen, and independently by Kato, even for M_1-spaces. However, in 16. the following partial justification of Nagata's conjecture was given: Every ground model has a CCC generic extension such that every regular, first countable M-space from the ground model satisfies Nagata's conjecture in the extension. The proof is based on the fact, interesting in itself, that every regular M_1-space from the ground model ha an M_1 „countably compactification" in the generic extension.

– **scattered spaces**

We continued the investigation of the cardinal sequences of compact scattered spaces (or equivalently: superatomic Boolean algebras) that occupied a central place in our earlier projects, and we made significant progress in the difficult task of characterizing these sequences. The cardinal sequence $\mathrm{SEQ}(X)$ of a compact scattered space X is the sequence of sizes of its infinite Cantor-Bendixson levels, taken in their natural well-order. We let $\mathcal{C}(\alpha)$ denote the class of all cardinal sequences of length α associated with compact scattered spaces and

$$\mathcal{C}_\lambda(\alpha) = \{f \in \mathcal{C}(\alpha) : f(0) = \lambda = \min[f(\beta) : \beta < \alpha]\}.$$

It is known that we get every member of $\mathcal{C}(\alpha)$ as a finite concatenation of the sequences from the subclasses $\mathcal{C}_\lambda(\alpha)$.

A (locally compact scattered = LCS) space X is called $\mathcal{C}_\lambda(\delta)$-*universal* in 37. if, on one hand, $\mathrm{SEQ}(X) \in \mathcal{C}_\lambda(\delta)$ and, on the other, for each sequence $s \in \mathcal{C}_\lambda(\delta)$ there is an open subspace Y of X with $\mathrm{SEQ}(Y) = s$. We showed there the following:

- there is a $\mathcal{C}_\omega(\omega_1)$-universal space,
- under CH there is a $\mathcal{C}_\omega(\delta)$-universal space for every $\delta < \omega_2$,
- under GCH for every infinite cardinal λ and every ordinal $\delta < \omega_2$ there is a $\mathcal{C}_\lambda(\delta)$-universal space,
- the existence of a $C_\omega(\omega_2)$-universal space is consistent.

As a consequence, the following is consistent: $2^\omega = \omega_2$ and $\mathcal{C}_\omega(\omega_2)$ is as large as possible, i.e. $\mathcal{C}_\omega(\omega_2) = \{s \in {}^{\omega_2}\{\omega, \omega_1, \omega_2\} : s(0) = \omega\}$.

The natural idea to prove the last result is to do some iterated forcing in such a way that in each step we add a space X_f to the intermediate model with cardinal sequence f for some $f \in {}^{\omega_2}\{\omega, \omega_1, \omega_2\}$. Since in each step we want to imitate the proof of Baumgartner and Shelah we try to use CCC iterands. However, in this case in each step we introduce new subsets of ω, and the length of the iteration is at least $|{}^{\omega_2}\{\omega, \omega_1, \omega_2\}| = \omega_3$, hence in the final model $2^\omega \geq \omega_3$.

That is why universal spaces came into the picture. A $\mathcal{C}_\omega(\omega_2)$-universal space may have cardinality ω_2, which gave us hope that it could be obtained by forcing with a single CCC poset P of cardinality ω_2, allowing us to get a generic extension with $2^\omega = \omega_2$.

In 39. we obtained some further results on the possible cardinal sequences of arbitrary length under GCH For any cardinal λ and ordinal $\delta < \lambda^{++}$ we define $\mathcal{D}_\lambda(\delta)$ as follows: for $\lambda = \omega$

$$\mathcal{D}_\omega(\delta) = \{s \in {}^{\delta}\{\omega, \omega_1\} : s(0) = \omega\},$$

and for $\lambda > \omega$

$$\mathcal{D}_\lambda(\delta) = \{s \in {}^{\delta}\{\lambda, \lambda^+\} : s(0) = \lambda, s^{-1}\{\lambda\} \text{ is } < \lambda\text{-closed and successor-closed in } \delta\}.$$

We have shown in our earlier work that if GCH holds and κ is regular, then $\mathcal{C}_\kappa(\delta) \subseteq \mathcal{D}_\kappa(\delta)$. In 39. we proved the following result:

If κ is an uncountable regular cardinal with $\kappa^{<\kappa} = \kappa$ and $2^\kappa = \kappa^+$ then for each $\delta < \kappa^{++}$ there is a κ-complete κ^+-c.c poset P of cardinality κ^+ such that in V^P

$$\mathcal{C}_\kappa(\delta) = \mathcal{D}_\kappa(\delta)$$

and there is a $\mathcal{C}_\kappa(\delta)$-universal LCS space.

As a consequence of this theorem we get that under GCH for any sequence f of regular cardinals of length α that is a potential member of $\mathcal{C}(\alpha)$, that is f may be appropriately obtained as the concatenation of sequences from finitely many $\mathcal{D}_\lambda$'s, can be made by a cardinal and GCH preserving forcing actually a member of $\mathcal{C}(\alpha)$.

Roitman proved the consistency of $\langle\omega\rangle_{\omega_1} {}^\frown \langle\omega_2\rangle \in \mathcal{C}(\omega_1+1)$. In 38. we generalized this by proving the following theorem.

Assume that κ, λ are infinite cardinals such that $\kappa^{+++} \leq \lambda$, $\kappa^{<\kappa} = \kappa$ and $2^\kappa = \kappa^+$. Then for any ordinal η with $\kappa^+ \leq \eta < \kappa^{++}$ and $cf(\eta) = \kappa^+$, we have $\langle\kappa\rangle_\eta {}^\frown \langle\lambda\rangle \in \mathcal{C}(\eta+1)$ in some cardinal-preserving generic extension.

In particular, both $\langle\omega\rangle_{\omega_1} {}^\frown \langle\omega_3\rangle \in \mathcal{C}(\omega+1)$ and $\langle\omega_1\rangle_{\omega_2} {}^\frown \langle\omega_4\rangle \in \mathcal{C}(\omega_1+1)$ are consistent. Just until recently, it looked hopeless to obtain the consistency of results like these. Another recent and quite unexpected result is the following theorem from 43. about cardinal sequences of length ω_2 (but this time in models that violate CH):

If GCH holds and $\lambda \geq \omega_2$ is regular then, in some cardinal preserving generic extension, we have $2^\omega = \lambda$ and every sequence $\langle\sigma_\alpha : \alpha < \omega_2\rangle \in \mathcal{C}(\omega_2)$ whenever $\omega \leq \sigma_\alpha \leq \lambda$.

An *open neighbourhood assignment* for a space X is a function η from X to the topology of X such that $x \in \eta(x)$ for every $x \in X$. We say that X is a *D-space*, if for every open neighbourhood assignment η for X there is a closed discrete subset D of X such that $\bigcup\{\eta(y) : y \in D\} = X$. Recently there has been a lot of interest in D spaces and in particular their additivity properties.

In 25. we show that a finite union of subparacompact scattered spaces is always a D space. This result can not be extended to countable unions, since it is known that there is a regular non-D space which is a countable union of paracompact scattered spaces. We also show that every countable union of regular, Lindelöf, $\mathcal{C}$-scattered spaces has the D-property and that locally finite unions of regular Lindelöf $\mathcal{C}$-scattered spaces are D.

cardinal functions

A celebrated result of Shapirovskii says that every every compactum of tightness κ has a π-base of order at most κ, in particular, a countably tight compactum has a point-countable π-base. Tkachuk noticed that under CH the latter special case remains valid if one replaces compact with Lindelöf but strengthens countably tight to first countable (he only considered Tychonov spaces). However, he did not know what happens if CH is not assumed. In fact, he did not even have a consistent example of any first countable space which did not possess a point-countable π-base. In 4. we settled a number of Tkachuk's problems concerning this. First of all, we gave ZFC examples of first countable Tychonov spaces whose all π-bases have order bigger than any previously given cardinal. However, our smallest ZFC example for a first countable space with no point-countable π-base has cardinality $\aleph_{\omega+1}$ and the best known ZFC lower bound is ω_2, so the problem is not completely settled. We do have *consistent* examples of even hereditarily Lindelöf first countable spaces of size ω_2 with no point-countable π-base which shows that the assumption of CH cannot be omitted in Tkachuk's result.

While the above results pointed toward the necessity of assuming compactness in Shapirovskii's above result, our main theorem in 12. shows that this is actually not so. Arhangelskii noticed that, by another result of Shapirovskii, all continuous images of countably tight compacta have countable π-character, and called this property countable *projective π-character*. He also noticed that compacta of countable projective π-character and having ω_1 as a caliber are separable, which, of course follows from having a point-countable π-base. Now, what we, quite unexpectedly, proved in 12. is that any Tychonov space of countable projective π-character has a point-countable π-base, or generally, any Tychonov space of projective π-character κ has a π-base of order at most κ. We also answered a question of Arhangelskii by showing that there are even compact spaces of countable projective π-character that are not countably tight, so ironically, our result is sharper than Shapirovskii's even among compacta.

Undoubtedly, the most significant recent advance in set-theoretic topology was J. Moore's ZFC example (in 2005) of an L-space, that is of a regular, hereditarily Lindelöf (HL) space of uncountable density. This naturally led to the problem: What cardinals could consistently occur as densities of such spaces? For regular cardinals we answered this in our previous OTKA project but the case of singular cardinals of uncountable cofinality turned out to be much harder. In 11. we could almost completely settle this problem by showing that for every ω-inaccessible cardinal λ, i.e. such that $\mu < \lambda$ implies $\mu^\omega < \lambda$, there is a CCC (hence cofinality and cardinality preserving) generic extension of the ground model in which λ is the density of a regular HL space.

We return here to 14. in which we answered several other questions of Tkachuk that concerned spaces more general than compacta. We proved there:

- For every T_1-space X the power $X^{d(X)}$ is d-separable.
- There is a 0-dimensional T_2-space whose ω_2nd power is d-separable but its ω_1st power is not.
- There is a 0-dimensional T_2-space X for which the function space $C_p(X)$ is not d-separable.

resolvability

A topological space X is called κ-resolvable if it contains κ disjoint dense subsets, and maximally resolvable if it is $\Delta(X)$-resolvable where $\Delta(X)$ is the smallest size of a non-empty open set in X. Both metric spaces and linearly ordered spaces are known to be maximally resolvable, and monotonically normal (MN) spaces form a class that includes them both. Thus it seems natural to raise the question if MN spaces are maximally resolvable. We investigated this problem in 11. and found some interesting and unexpected results:

(1) Every dense-in-itself MN space is ω-resolvable.
(2) If κ is a measurable cardinal then there is a MN space X with $\Delta(X) = \kappa$ which is not ω_1-resolvable.
(3) Every MN space of cardinality $< \aleph_\omega$ is maximally resolvable.
(4) From a supercompact cardinal we get the consistency of a MN space X with $|X| = \Delta(X) = \aleph_\omega$ that is not ω_2-resolvable.

The connection of the harmless looking topological problem and deep set-theory comes from a new class of MN spaces discovered in 12.. These spaces, we call them filtration spaces, are obtained on certain trees with the help of ultrafilters and their resolvability properties depend on the descendingly completeness properties of the ultrafilters used in their construction. We are presently working on the problems left open by the above results, for instance to see if a MN space that is not maximally resolvable exists in ZFC.

A topological space X is called *almost* κ-resolvable if it contains κ dense subsets that are almost disjoint in the sense that any two have nowhere dense intersection. Combining our earlier general method of $\mathcal{D}$- forced spaces, that was developed for the construction of spaces with various resolvability properties, with some new ideas we solved in 24. a problem raised by Comfort and Hu, resp. by Pavlov:

For every cardinal κ there is a 0-dimensional T_2-space X with $\Delta(X) = \kappa$ that is almost 2^κ-resolvable but not ω_1-resolvable.

Note that neither of the two values can be improved here, i.e. 2^κ cannot be increased and ω_1 cannot be decreased. This is trivial for the first and almost trivial for the second because any almost ω-resolvable space is already ω-resolvable.

II. Descriptive set theory

A *hull* of $A \subset [0,1]$ is a set H containing A such that $\lambda(H) = \lambda(A)$, where λ is the Lebesgue measure. We investigated in 19. all four versions of the following problem. Does there exist a monotone (wrt. inclusion) map that assigns a Borel/G_δ hull to every negligible/measurable subset of $[0,1]$? Three versions turned out to be independent of ZFC, while in the fourth case we could only prove that the non-existence of a monotone G_δ hull operation for all measurable sets is consistent. It remains open whether existence here is also consistent. We also answered a question of Z. Gyenes and D. Pálvölgyi which asked if monotone hulls can be defined for every chain (wrt. inclusion) of measurable sets. This line of research was very recently taken up by Roslanowski and Shelah.

In connection with the investigations of 19. we raised the problem if it is possible to give a represantation of all Borel sets as countable unions of simpler sets in a monotone way. This was answered negatively in 28. with the help of the theory of topologized Hurewicz test sets. This theory was introduced by Mátrai in his PhD thesis and then further developed in 6. with the aim of finding Hurewicz test sets for generalized separation and reduction properties of Borel sets. In 7. Mátrai extended the theory of topologized Hurewicz test sets from the Borel hierarchy to the full difference hierarchy. This enabled him to prove that the classes of the full difference hierarchy are closed under certain transfinite unions. This closure property was previously known only for the Borel classes.

The study of definable (e.g. Borel, analytic, etc.) ideals has been a very active field of research of descriptive set theory recently. In 40. a 20 year old conjecture of Kechris was refuted by producing a G_δ σ-ideal in the compact subsets of the Cantor set that covers the whole Cantor set but does not contain all the compact subsets of any dense G_δ subset of the Cantor set. This new type of ideal has since then found many other applications as well.

In 21. we presented a a number of new results about analytic P-ideals on ω: For any ideal $\mathcal{I}$ on ω we let $\mathfrak{a}(\mathcal{I})$ (resp. $\bar{\mathfrak{a}}(\mathcal{I})$) denote the minimum cardinality of a maximal infinite (resp. uncountable) $\mathcal{I}$-almost disjoint subfamily of $[\omega]^\omega$. We showed that $\mathfrak{a}(\mathcal{I}_h) > \omega$ if $\mathcal{I}_h$ is any summable ideal but $\mathfrak{a}(\mathcal{Z}_{\vec{\mu}}) = \omega$ for any tall density ideal $\mathcal{Z}_{\vec{\mu}}$, including the density zero ideal $\mathcal{Z}$. Moreover, we have $\mathfrak{b} \le \bar{\mathfrak{a}}(\mathcal{I})$ for any analytic P-ideal $\mathcal{I}$ and $\bar{\mathfrak{a}}(\mathcal{Z}_{\vec{\mu}}) \le \mathfrak{a}$ for any density ideal $\mathcal{Z}_{\vec{\mu}}$. For any ideal $\mathcal{I}$ on ω, $\mathfrak{b}_\mathcal{I}$ and $\mathfrak{d}_\mathcal{I}$ are the unbounding and dominating numbers of $\langle \omega^\omega, \le_\mathcal{I} \rangle$, where $f \le_\mathcal{I} g$ iff $\{n \in \omega : f(n) > g(n)\} \in \mathcal{I}$. We proved that $\mathfrak{b}_\mathcal{I} = \mathfrak{b}$ and $\mathfrak{d}_\mathcal{I} = \mathfrak{d}$ whenever $\mathcal{I}$ is an analytic P-ideal.

For an ideal $\mathcal{I}$ on ω the forcing $\mathbb{P}$ is $\mathcal{I}$-*bounding* iff $\forall x \in \mathcal{I} \cap V^\mathbb{P}\ \exists y \in \mathcal{I} \cap V\ x \subseteq y$ and $\mathcal{I}$-*dominating* iff $\exists y \in \mathcal{I} \cap V^\mathbb{P}\ \forall x \in \mathcal{I} \cap V\ x \subseteq^* y$. For an analytic P-ideal $\mathcal{I}$ if a forcing $\mathbb{P}$ has the Sacks property then it is $\mathcal{I}$-bounding; if $\mathcal{I}$ is also tall then the property $\mathcal{I}$-bounding/$\mathcal{I}$-dominating implies ω^ω-bounding/adding dominating reals and the converses of these two implications fail. For the density zero ideal $\mathcal{Z}$ we can prove more: (i) $\mathbb{P}$ is $\mathcal{Z}$-bounding iff it has the Sacks property, (ii) if $\mathbb{P}$ adds a slalom capturing all ground model reals then $\mathbb{P}$ is $\mathcal{Z}$-dominating.

III. Combinatorics

A coloring of a set-system $\mathcal{A}$ (formally, a map f defined on $\cup\mathcal{A}$) is called *conflict free* if every member of $A \in \mathcal{A}$ has a point whose color differs from the color of any other point in A. The *conflict free chromatic number* $\chi_{\mathrm{CF}}(\mathcal{A})$ of $\mathcal{A}$ is the smallest ρ for which $\mathcal{A}$ admits a conflict free coloring with ρ colors. Clearly, if all elements of $\mathcal{A}$ have size > 1 (that we always assume) then no member of A is monochromatic for a conflict free coloring, hence the chromatic number $\chi(\mathcal{A}) \le \chi_{\mathrm{CF}}(\mathcal{A})$.

$\mathcal{A}$ is a (λ, κ, μ)-system if $|\mathcal{A}| = \lambda$, $|A| = \kappa$ for all $A \in \mathcal{A}$, and $\mathcal{A}$ is μ-almost disjoint, i.e. $|A \cap A'| < \mu$ for distinct $A, A' \in \mathcal{A}$. Erdős and Hajnal investigated the chromatic numbers of (λ, κ, μ)-systems in the 60's and our aim in 35. was to run a parallel study of

$$\chi_{\mathrm{CF}}(\lambda, \kappa, \mu) = \sup\{\chi_{\mathrm{CF}}(\mathcal{A}) : \mathcal{A} \text{ is a } (\lambda, \kappa, \mu)\text{-system}\}$$

for $\lambda \ge \kappa \ge \mu$, actually restricting ourselves to $\lambda \ge \omega$ and $\mu \le \omega$. It turned out that the three cases 1.) $\omega > \kappa \ge \mu$, 2.) $\kappa \ge \omega > \mu$, and 3.) $\omega = \mu$ require very different methods. Here is a list of our main results:

(1) for any limit cardinal κ (or $\kappa = \omega$) and integers $n \ge 0$, $k > 0$, GCH implies

$$\chi_{\mathrm{CF}}(\kappa^{+n}, t, k+1) = \begin{cases} \kappa^{+(n+1-i)} & \text{if } \ i \cdot k < t \le (i+1) \cdot k\,,\ \ i = 1, ..., n; \\ \kappa & \text{if } \ (n+1) \cdot k < t\ ; \end{cases}$$

(2) if $\lambda \ge \kappa \ge \omega > d > 1$, then $\lambda < \kappa^{+\omega}$ implies $\chi_{\mathrm{CF}}(\lambda, \kappa, d) < \omega$ and $\lambda \ge \beth_\omega(\kappa)$ implies $\chi_{\mathrm{CF}}(\lambda, \kappa, d) = \omega$;

(3) GCH implies $\chi_{\mathrm{CF}}(\lambda, \kappa, \omega) \le \omega_2$ for $\lambda \ge \kappa \ge \omega_2$ and V=L implies $\chi_{\mathrm{CF}}(\lambda, \kappa, \omega) \le \omega_1$ for $\lambda \ge \kappa \ge \omega_1$;

(4) the existence of a supercompact cardinal implies the consistency of GCH plus $\chi_{\mathrm{CF}}(\aleph_{\omega+1}, \omega_1, \omega) = \aleph_{\omega+1}$ and $\chi_{\mathrm{CF}}(\aleph_{\omega+1}, \omega_n, \omega) = \omega_2$ for $2 \le n \le \omega$;

(5) CH implies $\chi_{\mathrm{CF}}(\omega_1, \omega, \omega) = \chi_{\mathrm{CF}}(\omega_1, \omega_1, \omega) = \omega_1$, while MA_{ω_1} implies $\chi_{\mathrm{CF}}(\omega_1, \omega, \omega) = \chi_{\mathrm{CF}}(\omega_1, \omega_1, \omega) = \omega$.

The *additivity spectrum* $ADD(\mathcal{I})$ of an ideal $\mathcal{I}$ is the set of all regular cardinals κ such that there is an increasing chain $\{A_\alpha : \alpha < \kappa\} \subset \mathcal{I}$ with $\cup_{\alpha<\kappa} A_\alpha \notin \mathcal{I}$. In 43. we investigated when a set of regular cardinals can be the additivity spectrum of certain ideals. Assume that $\mathcal{I}$ is either the σ-ideal generated by the compact subsets of the Baire space ω^ω or the ideal of null sets. We show that if A is a non-empty progressive set of uncountable regular cardinals and $\mathrm{pcf}(A) = A$, then $ADD(\mathcal{I}) = A$ in some CCC generic extension. We also show that if $A \subset ADD(\mathcal{I})$ is countable then $\mathrm{pcf}(A) \subset ADD(\mathcal{I})$. So for a non-empty countable set A of uncountable regular cardinals we have $A = ADD(\mathcal{I})$ in some CCC generic extension iff $\mathrm{pcf}(A) = A$.

The following problem was investigated in 18.: Let X be a set, κ a cardinal number, and $\mathcal{H}$ a family that covers each $x \in X$ at least κ times. Under what assumptions can we partition $\mathcal{H}$ into κ many subcovers? Equivalently, under what assumptions can we colour $\mathcal{H}$ by κ many colours so that for each $x \in X$ and each colour c there exists $H \in \mathcal{H}$ of colour c containing x? The assumptions we study may come from descriptive set theory, e.g. that $\mathcal{H}$ consists of open, closed, compact, G_δ, et. sets, or can be geometric: convex subsets of $\mathbb{R}^n$, or intervals in a linearly ordered set, or we can make various restrictions on the cardinality of X, $\mathcal{H}$ or the elements of $\mathcal{H}$. Besides numerous positive and negative ZFC results many of the questions turned out to be independent of ZFC.

Let $\mathbb{G}$ (resp. $\mathbb{D}$) denote the partially ordered set of homomorphism classes of finite undirected (resp. directed) graphs, ordered by the homomorphism relation. Order theoretic properties of $\mathbb{G}$ and $\mathbb{D}$ have been studied extensively, and have interesting connections with familiar graph properties and parameters. In particular, the notion of a duality is closely related to the idea of splitting maximal antichains in them. In 2. we constructed both splitting and non-splitting infinite maximal antichains in both $\mathbb{G}$ and in $\mathbb{D}$. The splitting maximal antichains give infinite versions of dualities for graphs and for directed graphs. In 32. we studied generalized duality pairs in $\mathbb{D}$ and gave a new, short proof for the Foniok - Nešetřil - Tardif theorem that characterizes all finite-finite duality pairs in $\mathbb{D}$. We also showed that there is no finite-infinite duality pairs of antichains in $\mathbb{D}$.

An independent set A of vertices of a directed graph is called *quasi-kernel* (resp. *quasi-sink*) iff for each vertex v there is a path of length at most 2 from A to v (resp. from v to A). Chvátal and Lovász proved that every finite directed graph has a quasi-kernel. The plain generalization of this to infinite directed graphs fails, even for tournaments, as is shown by $(\mathbb{Z}, <)$, where $\mathbb{Z}$ is the set of the integers and (x, y) is an edge iff $x < y$. However, in 20. we formulated and studied the following conjecture: for any directed graph $G = (V, E)$ there is a a partition (V_0, V_1) of the vertex set V such that the induced subgraph $G[V_0]$ has a quasi-kernel and the induced subgraph $G[V_1]$ has a quasi-sink. Although the conjecture remains open, we could proved it for a number of classes of infinite directed graphs.

IV. Real analysis and measure theory

Let $\mathbb{R}^{\mathbb{R}}$ denote the set of real valued functions defined on the real line. A map $D : \mathbb{R}^{\mathbb{R}} \to \mathbb{R}^{\mathbb{R}}$ is said to be a *difference operator*, if there are real numbers a_i, b_i ($i = 1, \dots, n$) such that $(Df)(x) = \sum_{i=1}^{n} a_i f(x + b_i)$ for every $f \in \mathbb{R}^{\mathbb{R}}$ and $x \in \mathbb{R}$. By a *system of difference equations* we mean a set of equations $S = \{D_i f = g_i : i \in I\}$, where I is an arbitrary set of indices, D_i is a difference operator and g_i is a given function for every $i \in I$, and f is the unknown function. One can prove that a system S is solvable if and only if every finite subsystem of S is solvable. However, if we look for solutions belonging to a given class of functions, then the analogous statement is no longer true. For example, there exists a system S such that every finite subsystem of S has a solution which is a trigonometric polynomial, but S has no such solution; in fact, S has no measurable solutions. This phenomenon motivates the following definition. Let $\mathcal{F}$ be a class of functions. The *solvability cardinal* $\mathrm{sc}(\mathcal{F})$ of $\mathcal{F}$ is the smallest cardinal number κ such that whenever S is a system of difference equations and each subsystem of S of cardinality less than κ has a solution in $\mathcal{F}$, then S itself has a solution in $\mathcal{F}$. In 3. we determined the solvability cardinals of most function classes that occur in analysis. As it turned out, the behaviour of $\mathrm{sc}(\mathcal{F})$ is rather erratic. For example, $\mathrm{sc}(\text{polynomials}) = 3$ but $\mathrm{sc}(\text{trigonometric polynomials}) = \omega_1$, $\mathrm{sc}(\{f : f \text{ is continuous}\}) = \omega_1$ but $\mathrm{sc}(\{f : f \text{ is Darboux}\}) = (2^{\omega})^{+}$, and $\mathrm{sc}(\mathbb{R}^{\mathbb{R}}) = \omega$. We also consistently determined the solvability cardinals of the classes of Borel, Lebesgue and Baire measurable functions, and gave some partial answers for the Baire class 1 and Baire class α functions.

A function $f : \mathbb{R}^n \to \mathbb{R}$ is called *vertically rigid* if $graph(cf)$ is isometric to $graph(f)$ for all $c \neq 0$. In 8. we settled Janković's conjecture by showing that a continuous function $f : \mathbb{R} \to \mathbb{R}$ is vertically rigid if and only if it is of the form $a + bx$ or $a + be^{kx}$ $(a, b, k \in \mathbb{R})$. In 18. we proved that a continuous function of two variables $f : \mathbb{R}^2 \to \mathbb{R}$ is vertically rigid if and only if after a suitable rotation around the z-axis $f(x, y)$ is of the form $a + bx + dy$, $a + s(y)e^{kx}$ or $a + be^{kx} + dy$ $(a, b, d, k \in \mathbb{R},\ k \neq 0,\ s : \mathbb{R} \to \mathbb{R}$ continuous). The problem of characterizing the vertically rigid continuous functions in more than two variables remains open.

A very fashionable topic of geometric measure theory was the subject of 31.: How stable is the „size" (e.g. the natural measure or Hausdorff measure) of the intersection of two copies of a fractal set under perturbations or other transformations. We obtain instability results stating that the measure of the intersection is separated from the measure of one copy. We also obtain results stating that the intersection is of positive measure if and only if it contains a relative open set. As an application we also obtain isometry (or at least translation) invariant measures of $\mathbb{R}^d$ such that the measure of the given self-similar or self-affine set is one.